The Casimir effect: a force from nothing

Giuseppe Bimonte
Physics Department
Università di Napoli Federico II-ITALY
INFN- Sezione di Napoli
Overview

- What is the Casimir effect?
- Old and new experiments
- The Casimir effect and MEMS
- The Casimir effect in superconducting cavities
In brief

The Casimir effect is a MACROSCOPIC force originating from QUANTUM VACUUM fluctuations of the electromagnetic field in constrained geometries.

Quantum fluctuating dipole moments inside atoms and molecules are at the origin of familiar van der Waals forces of chemistry (colloids).

When retardation effects, due to finite speed of light, are considered one passes smoothly from the van der Waals regime (short-distance non-retarded regime) to the Casimir-Polder regime (long-distance, retarded regime).

The Casimir force is a van der Waals interaction between MACROSCOPIC Bodies at separations ranging from a few nanometers to (a few) microns.
Radiation pressure is well known since a long time.

\[ P_{\text{rad}} = 2 \frac{W}{c} \]

Radiation pressure on a mirror placed inside an oven at temperature \( T \)

\[ P_T = \frac{\pi^2 (k_B T)^4}{45 (\hbar c)^3} \]

At room temperature (\( T=300 \text{ K} \))

\[ P_T = 2.0 \times 10^{-10} \frac{N}{cm^2} \]

What if we have TWO mirrors at distance \( a \) ?
Two mirrors form a **CAVITY**

Only photons that interfere costructively upon multiple reflections off the mirrors can propagate inside the cavity.

This gives a spectrum of allowed resonant frequencies $\omega_n$:

$$\omega_{n,k_{\perp}}(a) = c \sqrt{k_{\perp}^2 + \left(\frac{\pi n}{a}\right)^2} \quad n=0,1,2...$$

If $\frac{\hbar c}{a} >> k_B T$ no resonant modes can be thermally excited inside the cavity.

Therefore, each mirror only feels radiation pressure from **outside**, and we expect a NET force $F_T$ on either mirror, that pushes the mirrors towards each other:

$$F_T = P_T \times L^2 = \frac{\pi^2 \left(k_B T\right)^4}{45(\hbar c)^3} \times L^2$$
What is the force for larger separations between the mirrors?

Radiation enters the cavity and pushes also from inside. The force gets smaller

Sketch of computation:

The radiation inside the cavity can be regarded as an infinite set of HARMONIC OSCILLATORS with frequencies \( \omega_n \).

According to Planck, the free energy \( F_{osc} \) of an harmonic oscillator at temperature \( T \) is

\[
F_{osc}(\omega) = k_B T \log \left( 1 - \exp \left( \frac{\hbar \omega}{k_B T} \right) \right)
\]

The force on either mirror is

\[
F_T(a) = -\frac{\partial F}{\partial a}
\]

For large separations the force decreases rapidly:

\[
F_T = -\zeta(3) \frac{k_B T}{4\pi a^3} L^2 \quad \text{valid for} \quad a >> \frac{\hbar c}{k_B T}
\]
An interesting maritime analogy

S.L. Boersma Am. J. Phys. 64 (5) 1996

Fig. 1. Two ships roll heavily on a long swell and there is no more wind to damp their rolling. In this situation a strange force, “une certaine force attractive,” will pull the two ships toward each other. From P. C. Causseć: “the Mariners Album,” early 19th century.

Fig. 3. (a) Two ships at close quarters roll on a long swell. (b) They re-emit the absorbed power as secondary waves.
What happens to the force if we cool everything?

This looks like a stupid question to ask. At $T=0$ there is no radiation around and therefore there should be no force.

This conclusion is **WRONG** because according to QUANTUM MECHANICS it is impossible to freeze out completely electromagnetic fluctuations (Casimir 1948)
Heisenberg’s uncertainty principle implies that an oscillator in its ground state has an unescapable **zero-point energy** $E_0$

$$E_0 = \frac{1}{2} \hbar \omega$$

This is true all the same for the electromagnetic field!

There is no way to turn off zero-point quantum e.m. fluctuations inside the cavity by cooling it. Even at $T=0$ the cavity possesses a free-energy equal to the sum of the zero-point energies of all resonating e.m. modes.

Wait a minute! We have an infinite number of modes!

$$E_0(a) = \sum_{\text{All modes}} \frac{1}{2} \hbar \omega_{n,k} (a) = \infty$$

The zero-point energy of the cavity is INFINITE! This is NONSENSE!

True, BUT...
...we must SUBTRACT the zero-point energy $E_0$ of empty space (no mirrors)!
That too is INFINITE

Subtracting the two infinities gives a FINITE result (CASIMIR 1948)

$$E_{\text{Cas}}(a) = E_0(a) - E_0 = \infty - \infty = -\frac{\pi^2 \hbar c}{720a^3} \times L^2$$

The resulting Casimir force is

$$F_{\text{Cas}}(a) = \frac{\pi^2 \hbar c}{240a^4} \times L^2$$

$$F_{\text{cas}} = 0.13 \times 10^{-6} \left( \frac{1 \mu m}{a} \right)^4 N \quad \text{for } L^2=1 \text{ cm}^2$$

At 1 $\mu$m, $F_{\text{cas}}$ is 600 times larger than thermal pressure at room temperature!

The Casimir force is the dominant force at submicron distances. Its strength increases rapidly below one micron.

At 10 nm it equates the atmospheric pressure!
All this is for **ideal mirrors**. The theory for real materials was developed by Lifshitz long ago (1956). In combination with suitable approximations it permits to account for several factors like:

- finite skin depth of e.m. fields
- optical features of the mirrors
- temperature effects
- roughness of surfaces
Using the general theory one finds that:

The Casimir force can be repulsive if \( \varepsilon_1 > \varepsilon_2 > \varepsilon_3 \)

A Casimir torque arises between two birefringent plates

Presently explored directions:

- Dependence of the Casimir force on the **SHAPE** of the bodies
- Observation of **thermal** corrections
- Possibility of repulsive forces in vacuum (**METAMATERIALS**)
EXPERIMENTAL VERIFICATIONS
The plane-parallel configuration poses severe experimental problems (control of the parallelism, residual electric fields, accurate determination of separation...)

First experimental attempt
Sparnaay [Physica 24, 751 (1958)] at Philips labs

"The obtained results do not contradict Casimir's theoretical prediction..."
100% errors and also repulsive forces

The only other experiment in the plane-parallel configuration to date is G. Bressi, G. Carugno, R. Onofrio, G. Ruoso, Phys. Rev. Lett. 88, 041804 (2002) (experimental error 15%)
In order to avoid parallelism problems, other geometries are preferred, even though the Casimir force gets much smaller.


*Cylindrical surfaces of mica*

---

Fig. 1. Arrangement of crossed cylinders of mica.

Fig. 2. Logarithmic plot of critical jump distance $\Lambda$ against the parameter stiffness of beam/radius of cylinder ($c/E$). The results show a transition from non-retarded to retarded van der Waals forces at a separation of the order of 150 Å. For a separation above 200 Å the results agree well with those calculated from Lifshitz’s theory for retarded van der Waals forces using a theoretical value of $\Lambda = 0.87 \times 10^{-7}$. For separations below 100 Å the results agree with those calculated for non-retarded van der Waals forces using a value of the Hamaker constant $\Lambda = 10^{-8}$ erg.
Casimir force in a plane-spherical geometry

\[ F_C = -\frac{\pi^2 h c R}{720 d^3} \]

R= sphere radius
d= min. sphere-plate distance

From Mohideen-Roy (1998)

FIG. 1. Schematic diagram of the experimental setup. Application of voltage to the piezo results in the movement of the plate towards the sphere. The experiments were done at a pressure of 50 mTorr and at room temperature.

<table>
<thead>
<tr>
<th>Investigators</th>
<th>R</th>
<th>Range (μm)</th>
<th>Precision (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Van Blokland and Overbeek (1978)</td>
<td>1 m</td>
<td>0.13-0.67</td>
<td>25 at small distances</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>50 average</td>
</tr>
<tr>
<td>Lamoreaux (1997)</td>
<td>12.5 cm</td>
<td>0.6-6</td>
<td>5 at very small distance, larger elsewhere</td>
</tr>
<tr>
<td>Mohideen et al (1998)</td>
<td>200 μm</td>
<td>0.1-0.8</td>
<td>1</td>
</tr>
<tr>
<td>Chan et al (2001)</td>
<td>100 μm</td>
<td>0.075-2.2</td>
<td>1</td>
</tr>
</tbody>
</table>
Putting the Casimir Force at work

- **Tiny at Micron-scale**
  - It took 50 years to be verified!

- **Dominant at Nano-scale**
  - Strongest distance dependence

→ *unwanted effects: stiction*

→ *harnessed for good use: novel NEMS*
The Casimir force tilts the plate for sphere-plate distances less than 300 nm.

"...we demonstrated that when the separation between the surfaces is small, quantum effects...correctly describe the operation of our micromachined device. This could open new possibilities for novel actuation schemes in MEMS based on the Casimir force."
Lateral Casimir Force

• **Theoretical Prediction**

\[ F_{\text{lateral}} = \frac{2\pi \hbar c A a^2}{\lambda H^5} J \left( \frac{H}{\lambda} \right) \sin \left( \frac{2\pi b}{\lambda} \right) \]

Lateral Casimir Force

- Experimental Verification by AFM

Rack and Pinion (not yet realized)

- Powered by Quantum Vacuum
- Force transduction with no contact could potentially solve wear problem in nano-mechanical systems

The Aladin2 experiment

**SCIENTIFIC MOTIVATIONS**

- First direct measurement of the variation of Casimir energy in a rigid cavity.
- First demonstration of a phase transition influenced by vacuum fluctuations.

Aladin has been selected by INFN as a highligth experiment in 2006.
PARTICIPATING INSTITUTIONS

- INFN - Naples (Italy)

- IPHT (Institute for Physical High Technology) - Jena (Germany): U. Hubner, E. Il’Ichev

- Federico II University - Naples (Italy): E. Calloni, G. Bimonte, G. Esposito, L. Milano, L. Rosa, R. Vaglio

- Seconda Università di Napoli - Aversa (Italy): F. Tafuri, D. Born
We have realized two-layer systems, consisting of identical thin superconducting Al film, covered with an equal thickness of oxide. A cavity is obtained by covering some of the samples with a thick cap of a non-superconducting metal (Au).

1) The Casimir pressure on the outer layers and the free energy stored in the cavity depend on the reflective power of the layers.

2) The optical properties, in the microwave region, of a metal film change drastically when it becomes superconducting.

Therefore:

The Casimir pressure and free energy change when the state of the film passes from normal to superconducting.
The change in the Casimir pressure determined by the superconducting transition in the Al film is extremely small (of fractional order $10^{-8}$ or so) and practically unmeasurable even at the closest separations.

The reason is easy to understand: the main contribution to the Casimir energy arises from modes of energy $\hbar c/L \approx 10$ eV (for $L=20$ nm), while the transition to superconductivity affects the reflective power only in the microwave region, at the scale $k T_c \leq 10^{-4}$ eV (for $T_c \approx 1$ K).

A feasible alternative approach involves directly the variation $\Delta F_c$ of Casimir free energy across the transition:

$$\Delta F_c = F_c^{(n)} - F_c^{(s)} \neq 0$$

Indeed $\Delta F_c$ is expected to be positive, because, in the superconducting state, the film should be closer to behave as an ideal mirror than in the normal state, and so $F_c^{(s)}$ should be more negative than $F_c^{(n)}$. 
Is there a way to measure $\Delta F_c$?

$\Delta F_c$ can be measured by means of a comparative measurement of the (parallel) critical magnetic field $H_{c||}$ required to destroy the superconductivity of the three layer cavity, as compared to the critical field of the two-layer system (not forming a cavity).

Because of the Casimir energy $\Delta F_c$, the three-layer critical field is larger.

Since the effect depends on an energy scale (the film condensation energy $E_{\text{cond}}$) which is orders of magnitude smaller than typical Casimir energies $F_c$, even tiny variations $\Delta F_c$ of Casimir free energy give rise to measurable shifts $\delta H_c$. 
**Magnetic properties of superconductors**

- **Meissner effect**: they show perfect diamagnetism.
- **Superconductivity is destroyed by a critical magnetic field**.

The critical field depends on the shape of the sample and on the direction of the field. For a thick flat slab in a parallel field, it is called thermodynamical field and is denoted as $H_c$.

The value of $H_c$ is obtained by equating the magnetic energy (per unit volume) required to expel the magnetic field with the condensation energy (density) of the superconductor.

\[
\frac{H_c^2(T)}{8 \pi} = e_{\text{cond}}(T)
\]

( thick flat slab in parallel field)

\[
e_{\text{cond}}(T) = f_n(T) - f_s(T)
\]

$f_{n/s}(T)$: density of free energy at zero field in the n/s state

$H_c(T)$ follows an approximate Parabolic law

\[
H_c(T) = H_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]
\]
Superconducting film as a plate of a Casimir cavity

When the superconducting film is a plate of the cavity, the condensation energy $E_{\text{cond}}$ of the film is augmented by the difference $\Delta F_c$ among the Casimir free energies

$$\frac{1}{8\pi} \left( \frac{H_{c,(cav)}^2}{\rho} \right) V = E_{\text{cond}} + \Delta F_c$$

$\Delta F_c$ causes a shift of critical field $\delta H_c$:

$$\frac{\delta H_c}{H_c} \approx \frac{1}{2} \frac{\Delta F_c}{E_{\text{cond}}}$$

For an area $A=1 \text{ cm}^2$ and $L=10 \text{ nm}$

$$F_c \approx E_c = -\frac{\pi^2}{720} \frac{\hbar cA}{L^3} = -0.43 \text{ erg}$$

For an Al film with $A=1\text{cm}^2$, a thickness $D=10 \text{ nm}$, and for $T/T_c=0.995$:

$$E_{\text{cond}} = 4.4 \times 10^{-8} \text{ erg}$$

$F_c$ is 10 million times larger than $E_{\text{cond}}$!

So even a tiny fractional change of $F_c$ can be large compared with $E_{\text{cond}}$, and cause a measurable shift of critical field.
\( \delta T \approx 10 \div 5 \mu K \)
\( \delta H \approx 5 \text{ Gauss} \)
Low-field data for a bare film. The solid line is the extrapolation to low fields of a best fit of high field Data. The error bars are of $6 \mu K$.

Results of the measurements (preliminary).
The lower curve is the cavity, the upper one is for the bare film (Error bars are $6 \mu K$).
References:


Conclusions

- The Casimir force is nowadays a well tested reality (at the percent level)
- Open problem: making it repulsive in vacuum (metamaterials)
- Perspectives: engineering the Quantum Vacuum
Overview

- The problem of thermal Casimir effect in metallic cavities
- Johnson noise in conductors
- Interaction between two wires
- Role of finite size of conductors
- Conclusions
\[ F(a) = 2L^2 k_B T \sum_n \int \frac{d^2 k_\perp}{4\pi^2} \log \left[ 1 - \exp \left( \frac{\hbar \omega_n(k_\perp)}{k_B T} \right) \right] \]
Casimir effect

For an ideal cavity with perfectly reflecting mirrors (Casimir 1948):

\[ F(a) = -\frac{\pi^2 \hbar c}{240 a^4} \]

Modern experiments require considering a number of corrections:

- Surface roughness and shape of plates
- Finite conductivity of the plates
- Temperature of the plates

Surprisingly, for metallic plates, combined effect of temperature and finite conductivity raises severe problems.
Thermal corrections to the Casimir effect

The combined effect of finite conductivity and temperature, as given by Lifshitz theory, strongly depends on the model used for metal.

Energy correction factor $\eta_E$  

\[ \eta_E = \frac{E}{E_{id}} \]

Thermal correction for ideal plate

Thermal correction + plasma model

Plasma model, zero temperature

Drude model (with dissipation) + temperature

Nota bene: Dissipation produces a repulsive effect

FIG. 1. The energy correction factor for Au at 0 K (dotted line), Au at 300 K (solid line), Au at 300 K with the static transverse electric part incorrectly treated as in the perfect conductor case (circles), and finally the energy between perfect conductors evaluated at 300 K (dashed line). We have, as a comparison, also plotted the experimental energy of Ref. [5] (squares).

Experimental points from Lamoreaux,
Mathematical origin of the large thermal correction in dissipative metals

Using complex integration techniques, one can write the free energy as a sum over discrete imaginary frequencies \( \xi_l \) (Matsubara frequencies):

\[
F_E(a,T) = \frac{k_B T}{4\pi} \sum_{l=-\infty}^{\infty} \int_0^\infty k_1 dk_1 \{ \ln[1 - r_{||}^2(\xi_l, k_1) e^{-\frac{a q l}{2}}] + \ln[1 - r_{\perp}^2(\xi_l, k_1) e^{-\frac{a q l}{2}}] \},
\]

with

\[
\xi_l = 2\pi l k_B T / \hbar \\
l = \ldots, -2, -1, 0, 1, 2, \ldots
\]

For \( l = 0 \) (TE zero-mode), the contribution is:

\[
r_{\perp}^2(0, k_1) = \left( \frac{q_1 - k_1}{q_1 + k_1} \right)^2,
\]

\[
r_{||}^2(0, k_1) = \left( \frac{q_1}{q_1 + k_1} \right)^2.
\]

Dissipation:

\[
e_D(i \xi) = 1 + \frac{\omega_p^2}{\xi (\xi + \gamma)}.
\]

\[
e_p(i \xi) = 1 + \frac{\omega_p^2}{\xi^2}.
\]

TE zero-mode gives zero contribution:

\[
r_{||}^2(0, k_1) = 1, \quad r_{\perp}^2(0, k_1) = 0.
\]

TE zero-mode gives contribution:

\[
r_{||}^2(0, k_1) = 1, \quad r_{\perp}^2(0, k_1) = \left( \frac{ck_1 - \sqrt{\omega_p^2 + c^2 k_1^2}}{ck_1 + \sqrt{\omega_p^2 + c^2 k_1^2}} \right)^2.
\]
Δ\(P(a,T)\) depends on low frequencies from \(k_B T/\hbar\) (infrared) down to microwaves.

\(P_0(a,T)\) depends on \(T\) as a parameter and receives contributions from frequencies around \(\omega_c = c/2a\). Dissipation has little effect on \(P_0(a,T)\).

A deeper physical insight is achieved by separating the thermal correction from the contribution of zero-point fluctuations, and by looking at the spectrum along the real-frequency axis (Torgerson and Lamoreaux, 2004).

Split the Casimir Pressure \(P(a,T)\) as

\[
P(a, T) = P_0(a, T) + \Delta P(a, T)
\]

**Contribution of zero-point fluctuations**

\[
P_0(a, T) = -\frac{\hbar}{2\pi^2} \int_0^\infty d\omega \int_0^\infty dk_\perp k_\perp \text{Re} \left\{ k_z \sum_{\alpha=\text{TE,TM}} \left[ 1 - \frac{e^{-2ik_za}}{r_\alpha^{(1)} r_\alpha^{(2)}} \right]^{-1} \right\}
\]

\(P_0(a,T)\) depends on \(T\) as a parameter and receives contributions form frequencies around \(\omega_c = c/2a\).

**Contribution of Thermal radiation**

\[
\Delta P(a, T) = -\frac{\hbar}{\pi^2} \int_0^\infty d\omega \int_0^\infty dk_\perp k_\perp \frac{1}{\exp\left(\frac{\hbar\omega}{k_BT}\right) - 1} \text{Re} \left\{ k_z \sum_{\alpha=\text{TE,TM}} \left[ 1 - \frac{e^{-2ik_za}}{r_\alpha^{(1)} r_\alpha^{(2)}} \right]^{-1} \right\}
\]

\(\Delta P(a,T)\) depends on low frequencies from \(k_B T/\hbar\) (infrared) down to microwaves.
The large thermal correction arises from thermal evanescent waves with transverse electric polarization (TE EW)

Spectrum of the thermal correction
Torgerson and Lamoreaux (2004)

Contribution to $\Delta P(a,T)$ from TE Propagating waves
Dashed-line is for ideal metal

Contribution to $\Delta P(a,T)$ from TE Evanescent waves (TE EW)
For zero dissipation this is zero.

$$\omega = \gamma \left( \frac{\omega_c}{\Omega_P} \right)^2 = 6.4 \times 10^9 \text{ rad/s}$$

$$\omega_c = \frac{c}{2a}$$
characteristic frequency of the cavity

Solid line is for Au with $T=300$ K, $a=1$ μm.
Dashed-line is for ideal metal.
(from Torgerson and Lamoreaux, PRL 70 (2004) 047102)
The contribution from thermal EW

\[
\Delta P_{EW}(a, T) = \frac{\hbar}{\pi^2} \int_0^\infty d\omega \frac{1}{\exp\left(\frac{\hbar \omega}{k_B T}\right) - 1} \int_0^\infty dq q^2 \\
\times \sum_{\alpha=TE,TM} \text{Im} \left[ 1 - \frac{e^{2qa}}{r_{\alpha}^{(1)}(\omega, k_\perp) r_{\alpha}^{(2)}(\omega, k_\perp)} \right]^{-1}
\]

Distinctive features are

- It is always **repulsive**
- It does not vanish in the limit of vanishing dissipation
- It is **zero for strictly zero dissipation** (plasma model)
- For metals without impurities, it violates Nernst heat theorem (Bezerra et al. 2004)
- It involves low frequencies \( \omega \approx \gamma \left(\Omega_c/\Omega_p\right)^2 \), with \( \Omega_c = c/(2a) \)

Remark: for vanishing dissipation, the plasma model results are recovered smoothly in the propagating sector and in the TM evanescent sector.
Let us summarize the Physics of the problem

**Ideal mirrors at T=0**: cavity has standing modes
- $TM \ n=0,1,2,3...$
- $TE \ n=1,2...$
- Casimir Energy $\leftrightarrow$ Zero-point energy of cavity modes

**Ideal mirrors at T>0**: cavity modes get populated
- Free Energy receives contribution from thermal photons

**Finite plasma frequency** (any T): cavity modes penetrate walls a bit.
- Spectrum of modes broadens a bit (small thermal correction)

**Dissipation**:
- TE EW appear $\rightarrow$ Large thermal correction
What is the physics behind the large thermal TE EW contribution?

The sudden appearence of a new sector of e.m. fluctuations as soon as dissipation is turned on indicates that these fluctuations are related to a NEW physical phenomenon characteristic of conductors, that is absent when dissipation is zero.

Hint: the relevant low-frequency thermal TE EW consist of a continuous spectrum of fluctuating magnetic fields.

What produces these fields?

The Johnson-Nyquist currents (1928) inside the plates.

Physical Picture: Johnson currents in either plate induce (correlated) eddy currents in the other plate. Repulsion arises from the magnetic interaction between them.
a noisy resistor can be modelled by a noiseless resistor connected to either a e.m.f. or a current noise generator

\[
\langle E(\omega)E^*(\omega') \rangle = 4\pi R \frac{\hbar \omega}{\hbar \omega} e^{\frac{\hbar \omega}{k_B T}} - 1 \\
\langle I(\omega)I^*(\omega') \rangle = 4\pi \frac{1}{R} \frac{\hbar \omega}{\hbar \omega} e^{\frac{\hbar \omega}{k_B T}} - 1
\]

Nyquist (1928)
Thermal interaction between two nearby metallic wires

Two nearby metallic wires

The circuit equations (low-frequency approx.)

\[
\begin{align*}
\mathcal{L} \frac{di_1}{dt} + \mathcal{M}(\vec{a}) \frac{di_2}{dt} + R i_1 &= \mathcal{E}_1(t) \\
\mathcal{L} \frac{di_2}{dt} + \mathcal{M}(\vec{a}) \frac{di_1}{dt} + R i_2 &= \mathcal{E}_2(t)
\end{align*}
\]

Random e.m.f.

Wires self-inductance

Wires mutual inductance

\[
\langle \mathcal{E}_i(\omega) \mathcal{E}_j^*(\omega') \rangle = 4\pi R \frac{\hbar \omega}{\exp \left( \frac{\hbar \omega}{k_B T} \right) - 1} \delta(\omega - \omega') \delta_{ij}
\]

Force between the wires:

\[
\vec{F}_{12}(\vec{a}) = \langle i_1 i_2 \rangle \vec{\nabla}_a \mathcal{M}(\vec{a})
\]
Thermal interaction between two nearby noisy resistors

\[ \vec{F}_{12} = -k_B T H \vec{\nabla}_a (m^2) \]
\[ m = \mathcal{M}/\mathcal{L} \]
\[ \omega_R = R/\mathcal{L} \]

\[ H = \frac{1}{\pi} \int_0^\infty d\omega \omega E \left( \frac{\omega}{\omega_T} \right) \text{Im} \left[ (\omega_R - i\omega)^2 + \omega^2 m^2 \right]^{-1} \]

The force is always repulsive.

For vanishing R, the force does not vanish

\[ \lim_{R \to 0} \vec{F}_{12} = -k_B T f(m^2) \vec{\nabla}_a (m^2) \]

Free-energy

\[ \mathcal{F} = \frac{k_B T}{\pi} \int_0^\infty d\omega \frac{\omega}{\omega_T} E \left( \frac{\omega}{\omega_T} \right) \text{Im} \log \left[ 1 + \left( \frac{\omega m}{\omega_R - i\omega} \right)^2 \right] \]

For low T (in metals without impurities) R vanishes like \( T^2 \). Then

\[ \mathcal{F} \approx g(m^2) k_B T \]

\[ \lim_{T \to 0} S = -k_B g(m^2) \equiv S_0 < 0 \]

Nernst th. violated
Thermal interaction between two nearby noisy resistors

Summarizing: the electrodynamic interaction between Johnson and eddy currents gives rise to a force that:

- is repulsive
- does not vanish for $R \rightarrow 0$
- vanishes for strictly dissipationless wires $R=0$
- violates Nernst th. (for wires with no impurities)

**All this is as in the thermal Casimir effect**

supporting our Physical picture of the large thermal correction to the Casimir pressure, as originating from Johnson currents in the plates.
Question: have we missed anything?

Yes: a **finite** wire has end-points, where **charges build up**

In our circuit equations we should take account of this, by including a **capacitance**

\[
\begin{align*}
L \frac{di_1}{dt} + M(\vec{a}) \frac{di_2}{dt} + R_i \frac{Q_1}{C} = \mathcal{E}_1(t) \\
L \frac{di_2}{dt} + M(\vec{a}) \frac{di_1}{dt} + R_i \frac{Q_2}{C} = \mathcal{E}_2(t)
\end{align*}
\]

Capacitances act as **high-pass filters** and block low frequencies
Thermal interaction between two nearby noisy resistors

Inclusion of capacitances ensures that

- the force vanishes for vanishing \( R \)
- the entropy vanishes for \( T=0 \)

Indeed

\[
\mathcal{F} = -\frac{16\pi^5 m^2}{63} \left( \frac{k_B T}{\hbar \omega_C} \right)^6 \hbar \omega_R.
\]

\[
\omega_C = \frac{1}{\sqrt{\mathcal{L} C}}.
\]

There still is a range of \( T \) for which entropy is negative

Plot of the free energy (in units of \( \hbar \omega_C \)) as a function of \( t = k_B T/(\hbar \omega_C) \).
However: each wire being a RLC circuit, it possesses an associated free energy

\[ \mathcal{F}_{\text{self}} = k_B T \log[1 - \exp(-\hbar \omega_C / (k_B T))] \]

Inclusion of the wires self-entropy makes the total entropy of the system positive at all temperatures, while respecting Nernst th.

Inclusion of capacitances for the end-points gives a fully satisfactory picture
Are edge effects important in the case of bulk plates of size $L$?

**It depends**

Recall that typical size of current fluctuations is of order of plates separation $a$.

At room temperature, if $a \ll L$, edge effects are expected to be unimportant.

If $a \approx L$, as in MEMS, edge effects become important.

At low temperature, the problem is more complicated. **Spatial correlation** of Johnson currents (anomalous skin effect) suppress fluctuations at small scales. If correlations extend over distances comparable to $L$, edge effects become important. (work in progress)
Conclusions

- Johnson noise provides the physical explanation of the large thermal correction to the Casimir pressure in metallic cavities.

- Johnson noise exists only in conductors with dissipation. It is therefore explained why large thermal corrections are absent in models which neglect dissipation.

- Edge effects arising from finite size of the conductors resolve all thermodynamical inconsistencies in the case of wires.

- Edge effects are expected to be unimportant for closely spaced bulk plates at room temperature, but may be important for a resolution of thermodynamical inconsistencies at low temperature (work in progress).
TE EW and heat transfer

Thermal EW give the dominant contribution to radiative heat transfer between two metallic surfaces, separated by an empty gap, at submicron separations (Polder-van Hove (1971)).

The frequencies involved are same as in thermal corrections to the Casimir force. Therefore, heat transfer gives information on thermal TE EW (G. Bimonte, PRL 96 (2006) 160401).
The power $S$ of heat transfer

The contribution to $S$ from EW:

$$S_{EW} = \frac{\hbar}{\pi^2} \int_0^\infty d\omega \omega \left( \frac{1}{\exp(h\omega/k_BT_1) - 1} - \frac{1}{\exp(h\omega/k_BT_2) - 1} \right) \int_0^\infty dq \, q \times \sum_{\alpha = TE, TM} \frac{\text{Im} r_{\alpha}^{(1)}(\omega, k_\perp) \text{Im} r_{\alpha}^{(2)}(\omega, k_\perp)}{|1 - r_{\alpha}^{(1)}(\omega, k_\perp)r_{\alpha}^{(2)}(\omega, k_\perp)\exp(-2qa)|^2} e^{-2qa}.$$  

Note again that $S_{EW}$ vanishes if the reflection coefficients are real.

We have compared the powers $S$ of heat transfer implied by various models of dielectric functions and surface impedances, that are used to estimate the thermal Casimir force (Bimonte, G. Klimchitskaya and V.M. Mostepanenko (2006) submitted).
Models for the metal

- The Drude model (Lifshitz theory):

\[
\varepsilon_D = 1 - \frac{\Omega^2}{\omega(\omega + i\gamma)}
\]

- The surface impedance of the normal skin effect \(Z_N\):

\[
Z_N = (1-i)\sqrt{\frac{\omega}{8\pi\sigma_0}}
\]

- The surface impedance of the Drude model \(Z_D\):

\[
Z_D = \frac{1}{\sqrt{\varepsilon_D}}
\]

- A modified form of the surface impedance of infrared optics, including relaxation effects \(Z_t\):

\[
Z_t = -i\frac{\omega}{\sqrt{\omega_p^2 - \omega^2}} + Z_t'
\]
Modified expression of the infrared-optics impedance

\[ Z_t' = \begin{cases} 
B \sin \left( \frac{\pi \omega^2}{2 \beta^2} \right), & \omega \leq \beta \\
B, & \beta \leq \omega \leq 0.125 \text{ eV} \\
Y(\omega), & \omega \geq 0.125 \text{ eV}
\end{cases} \]

\( Y(\omega) \) stands for tabulated data, available for \( \omega > 0.125 \text{ eV} \)
We allowed \( 0.08 \text{ eV} < \beta < 0.125 \text{ eV} \)
Comparison of powers $S$ of heat transfer

$$\varepsilon_D = 1 - \frac{\Omega^2}{\omega (\omega + i\gamma)} \quad \text{(Lifshitz theory)}$$

$$Z_D = \frac{1}{\sqrt{\varepsilon_D}}$$

$$Z_N = (1-i) \sqrt{\frac{\omega}{8\pi\sigma_0}}$$

$Z_t$ optical data + extrapolation to low frequencies

$S$ (in erg cm$^{-2}$sec$^{-1}$)

separation in $\mu$m
Hargreaves measurements for chromium plates (1969)
The measurements from Hargreaves were compared with the theoretical expression implied by the Drude model, showing only qualitative agreement.

In Ref. 5, Hargreaves published measurements of radiative heat transfer between flat chromium bodies with a mean temperature of $T \approx 315$ K in the separation range $1 < d < 10$ μm. We find very good agreement with experiment as regards the shape of the curves and the critical distance below which the small-separation effect becomes noticeable.

The absolute values of the heat currents, though of the same order of magnitude, do not coincide, however, not even for $d = \infty$. It appears after examination of more recent, as yet unpublished, measurements by the same author that the discrepancy lies in a difference between bulk chromium (on which our calculations are based) and the chromium layers used in the experiments.
CONCLUSIONS

- The controversy on thermal Casimir effect for real metals originates from difficulties with thermal TE EW radiated by metal surfaces.

- Thermal TE EW are also probed by experiments on radiative heat transfer.

- It is important to have new experiments on heat transfer for the metals used in Casimir experiments.