From twisted classical mechanics to twisted quantum fields

Patrizia Vitale
Università di Napoli "Federico II"

with Paolo Aschieri and Fedele Lizzi

Problemi attuali di Fisica Teorica, Vietri 14 -19 marzo 2008

Plan of the talk

- Motivations
- Twist principle for deforming the differential geometry
- Classical Mechanics deformed Poisson brackets
- Classical field theory. deformed Poisson brackets
- Quantum Field theory deformed commutation rules
- Conclusions and perspectives

Motivation: Evidences from General Relativity, string theory and black hole physics.

QM + GR (DFR): If one tries to locate an event with a spatial accuracy comparable with Planck length, spacetime uncertainty relations emerge. As in QM, these are naturally implied by non-commuting coordinates,

$$[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}\Theta^{\mu\nu}$$

$\Theta^{\mu\nu}$ in general coordinate dependent. Its specific form qualifies the kind of noncommutativity.

Below Planck length the usual description of spacetime as a pseudo-Riemannian manifold locally modeled on Minkowski space is not adequate. It has been proposed (Connes) that it be described by a *Noncommutative Geometry*.
Two relevant issues:

- The formulation of General Relativity

- The quantization of field theories on noncommutative spacetime.

There are different proposals for this second issue, and different canonical commutation relations have been considered

We work in the deformation quantization context; noncommutativity is obtained by introducing the Moyal product on the algebra of smooth functions on spacetime

$$f\star g(x) = f(x)\,\mathrm{e}^{\frac{\mathrm{i}}{2}\theta^{\mu\nu}\frac{\overleftarrow{\partial}}{\partial x^\mu}\frac{\overrightarrow{\partial}}{\partial x^\nu}}g(x)$$

which implies

$$x^\mu\star x^\nu - x^\nu\star x^\mu = \mathrm{i}\theta^{\mu\nu}$$

$$\int f\star g = \int g\star f = \int f\cdot g$$

We choose θ to be a fundamental physical constant.

This means that spacetime (and phase-space) symmetries are not ordinary Lie symmetries but NC Hopf algebras [Chaichian et al, Wess et al].

NC spacetime geometry

- We consider $d+1$ dim spacetime with spatial Moyal noncommutativity

- We associate such noncommutativity to a twist operator

$$f \star g := \mu \circ \mathcal{F}^{-1}(f \otimes g) \ ,$$

with

$$\mathcal{F} = \mathrm{e}^{-\frac{\mathrm{i}}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\otimes\frac{\partial}{\partial x^\nu}}$$

with $\mu, \nu = 1, \ldots, d$.

Basic Notation

– $\frac{\partial}{\partial x^\mu}$ are globally defined vectorfields on $\mathbb{R}^d$ (infinitesimal translations).

– Ξ is the Lie algebra of vector fields

– $U\Xi$ its enveloping algebra

– $\mathcal{F} \in U\Xi \otimes U\Xi.$, with

$$\begin{aligned}
\mathcal{F} &= 1 \otimes 1 - \frac{\mathrm{i}}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu} \otimes \frac{\partial}{\partial x^\nu} - \frac{1}{4}\theta^{\mu\nu}\theta^{\rho\sigma}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\rho} \otimes \frac{\partial}{\partial x^\nu}\frac{\partial}{\partial x^\sigma} + \ldots \\
&= \mathsf{f}^\alpha \otimes \mathsf{f}_\alpha \\
\mathcal{F}^{-1} &= \bar{\mathsf{f}}^\alpha \otimes \bar{\mathsf{f}}_\alpha
\end{aligned}$$

– $\mathcal{R}$ is the universal $\mathcal{R}$-matrix

$$\mathcal{R} := \mathcal{F}_{21}\mathcal{F}^{-1}$$

with $\mathcal{F}_{21} = \mathrm{f}_\alpha \otimes \mathrm{f}^\alpha$. For Moyal $\mathcal{R} = \mathrm{e}^{-\mathrm{i}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\otimes\frac{\partial}{\partial x^\nu}}$

– we use the notation $\mathcal{R} = R^\alpha \otimes R_\alpha \quad , \quad \mathcal{R}^{-1} = \bar{R}^\alpha \otimes \bar{R}_\alpha$.

The $\mathcal{R}$-matrix measures the noncommutativity of the $\star$-product.

$$h \star g = \bar{R}^\alpha(g) \star \bar{R}_\alpha(h) \ .$$

The permutation group in noncommutative space is naturally represented by $\mathcal{R}$.

Twist principle

- The guiding principle is: every time we have a bilinear map

$$\mu : X \times Y \to Z$$

we combine this map with the action of the twist

$$\mu_\star := \mu \circ \mathcal{F}^{-1}$$

- from this principle we not only deduce a deformed differential geometry of spacetime, but also a deformed differential geometry of the phase-space of point mechanics and fields.

NC spacetime differential geometry

It is obtained by applying the twist principle to the algebras of vectorfields, 1-forms and tensorfields.

Vectorfields $\Xi_\star$. The product $\mu : \mathcal{A} \times \Xi \to \Xi$ between of functions on spacetime and vectorfields is deformed.

$$f \star V = \bar{\mathsf{f}}^\alpha(f)\bar{\mathsf{f}}_\alpha(V)$$

We have $f\star(g\star V) = (f\star g)\star V$,
$\Xi_\star$ is the space of vectorfields with this $\star$-multiplication. As vectorspaces $\Xi{=}\Xi_\star$, but Ξ is a $\mathcal{A}$-module while $\Xi_\star$ is a $\mathcal{A}_\star$-module.

For 1-forms and tensor fields we proceed in a similar way.

How they act on functions on spacetime

The $\star$-Lie derivative on the algebra of functions $\mathcal{A}_\star$ is obtained following the general prescription.

$$\mathcal{L}^\star_V(h) := \bar{\mathrm{f}}^\alpha(V)(\bar{\mathrm{f}}_\alpha(h))$$

The differential operator $\mathcal{L}^\star_V$ satisfies the deformed Leibniz rule

$$\mathcal{L}^\star_V(h\star g) = \mathcal{L}^\star_V(h)\star g + \bar{R}^\alpha(h)\star\mathcal{L}^\star_{\bar{R}_\alpha(V)}(g)$$

we can define an exterior derivative, a contraction ...

The Leibniz rule is consistent (and actually follows) from the coproduct rule

$$V \mapsto \Delta_\star V = V \otimes 1 + \bar{R}^\alpha \otimes \bar{R}_\alpha(V)$$

$\Xi_\star$ is a $\star$-Lie Algebra

In the commutative case the commutator of two vectorfields is again a vectorfield$\longrightarrow$ Lie algebra of vectorfields.

In this $\star$-deformed case the deformed bracket is obtained from the undeformed one via composition with the twist:

$$[\ ,\ ]_\star = [\ ,\ ] \circ \mathcal{F}^{-1}.$$

that is

$$[U, V]_\star := [\bar{\mathrm{f}}^\alpha(U), \bar{\mathrm{f}}_\alpha(V)]$$

Remarkably the rhs is a first order differential operator.

Therefore, in the presence of twisted noncommutativity, we replace the usual Lie algebra of vectorfields, Ξ, with $\Xi_\star$.

On functions

$$\mathcal{L}^\star_U\,\mathcal{L}^\star_V - \mathcal{L}^\star_{\bar{R}^\alpha(V)}\,\mathcal{L}^\star_{\bar{R}_\alpha(U)} = \mathcal{L}^\star_{[U,V]_\star}$$

The bracket $[\ \ ,\ \ ]_\star\ :\ \Xi_\star \times \Xi_\star \to \Xi_\star$ is a bilinear map and verifies the $\star$-antisymmetry and the $\star$-Jacobi identity

$$[U,V]_\star = -[\bar{R}^\alpha(V), \bar{R}_\alpha(U)]_\star\ .$$

$$[U,[V,Z]_\star]_\star = [[U,V]_\star, Z]_\star + [\bar{R}^\alpha(V), [\bar{R}_\alpha(U), Z]_\star]_\star\ .$$

A $\star$-Lie algebra is not a generic name for a deformation of a Lie algebra. It is a quantum Lie algebra of a quantum group.

$$U\Xi_\star$$

The $\star$-Lie algebra $\Xi_\star$ gives rise to the universal enveloping algebra $U\Xi_\star$ of sums of products of vectorfields, with the identification

$$U\star V - \bar{R}^\alpha(V)\star\bar{R}_\alpha(U) = [U,V]_\star$$

and coproduct

$$\Delta_\star(U) = U\otimes 1 + \bar{R}^\alpha \otimes \bar{R}_\alpha(U)$$

Classical Mechanics

Phase space geometry: Since momenta are commutative the twist lifts trivially to phase space

Poisson bracket: The standard one does not define a derivation of the new algebra of observables $\mathcal{A}_\star = C^\infty(M)_\star$

$$\{f, g \star h\} \neq \{f, g\} \star h + g \star \{f, h\}$$

We deform it according to our general prescription:

$$\{\ ,\ \}_\star := \{\ ,\ \} \circ \mathcal{F}^{-1}$$

with $\{\ ,\ \}$ the canonical one

$$\{f, g\}_\star := \{\bar{\mathrm{f}}^\alpha(f), \bar{\mathrm{f}}_\alpha(g)\} = \frac{\partial f}{\partial x^\ell} \star \frac{\partial g}{\partial p_\ell} - \frac{\partial f}{\partial p_\ell} \star \frac{\partial g}{\partial x^\ell} \ .$$

Properties:

$$\begin{aligned} \{f,g\}_\star &= -\{\bar{R}^\alpha(g), \bar{R}_\alpha(f)\}_\star \quad \mathcal{R}-antisymmetry \\ \{f, g\star h\}_\star &= \{f,g\}_\star \star h + \bar{R}^\alpha(g)\star\{\bar{R}_\alpha(f),h\}_\star \quad \star - Leibniz \\ \{f,\{g,h\}_\star\}_\star &= \{\{f,g\}_\star, h\}_\star + \{\bar{R}^\alpha(g), \{\bar{R}_\alpha(f),h\}_\star\}_\star \quad \star - Jacobi \end{aligned}$$

$\Rightarrow$ It defines $\star$-derivations.

$$\{f,\ \}_\star = \mathcal{L}^\star_{X_f}$$

with X_f the undeformed Hamiltonian vector field.
The Leibniz rule can be rewritten as

$$\mathcal{L}^\star_{X_f}(g\star h) = \mathcal{L}^\star_{X_f}(g)\star h + \bar{R}^\alpha(g)\star\mathcal{L}^\star_{X_{\bar{R}_\alpha(f)}}(h)$$

and is consistent (and actually follows) from the coproduct rule.

- *Hamiltonian vector fields are a $\star$-Lie subalgebra of the $\star$-Lie algebra of vectorfields*.

$$[X_f, X_g]_\star = X_{\{f,g\}_\star}$$

- These results can be generalized to a general twist $\mathcal{F}$ on an arbitray Poisson manifold M provided compatibility relations between twist and Poisson structure are satisfied.

Time Evolution, Constants of Motion, Symmetries

Time evolution

$$\dot{f} = -\mathcal{L}^\star_{X_H} f = -\{H, f\}_\star$$

The time evolution generator $X_H = \frac{\partial H}{\partial x^i}\frac{\partial}{\partial p_i} - \frac{\partial H}{\partial p_i}\frac{\partial}{\partial x^i}$ is the same as the undeformed one;

it is its action $\mathcal{L}^\star$ on functions that is deformed. In general $\{H, f\}_\star \neq \{H, f\}$

A constant of motion is a function Q on phase space that satisfies

$$\{H, Q\}_\star \equiv \mathcal{L}^*_{X_H} Q = 0$$

If

$$\{Q, H\}_\star \equiv \mathcal{L}^*_{X_Q} H = 0$$

we say that the Hamiltonian is invariant under the vectorfield X_Q

Since the $\star$-Poisson bracket is not antisymmetric these are are independent equations

this means that having constants of motion doesn't necessarily imply symmetries of the Hamiltonian. We are in trouble with Nöther theorem;

with an important exception:

for translation invariant Hamiltonians the time evolution equation as well as the notion of constant of motion are undeformed.
$\Rightarrow$
The concept of constant of motion and that of symmetry coincide.
Examples of translation invariant Hamiltonians include point particles Hamiltonians whose potential depends only on the relative distance of the point particles involved and field theories.

The $\star$-bracket $\{Q, Q'\}_\star$ of two constants of motion is a constant of motion. $\Rightarrow$

The subspace of Hamiltonian vector fields X_Q that $\star$-commute with X_H form a $\star$-Lie subalgebra of the $\star$-Lie algebra of Hamiltonian vectorfields: the $\star$-algebra of constants of motion.

NC Classical Field Theory in the Hamiltonian formalism

We generalize to the case of an infinite number of degrees of freedom.
Positions and momenta generalize to the fields $\Phi(x)$ and $\Pi(x)$ with $x \in \mathbb{R}^d$ ($\mathbb{R}^{d+1}$ being spacetime).
The algebra of observables, A is an algebra of functionals, i.e. functions on the function space N:

$$N = \mathrm{Maps}\,(\mathbb{R}^d \to \mathbb{R}^2)$$

$$\Psi \in N : x \to (\Phi(x), \Pi(x))$$

A typical Hamiltonian

$$H = \int d^d x \ \partial_i \Phi \star \partial^i \Phi + \Pi_i \star \Pi^i + V^\star(\Phi)$$

We define the Poisson bracket between the functionals $F, G \in \mathsf{A}$ to be

$$\{F, G\} = \int \mathsf{d}^d x \;\; \frac{\delta F}{\delta \Phi}\frac{\delta G}{\delta \Pi} - \frac{\delta F}{\delta \Pi}\frac{\delta G}{\delta \Phi}$$

$\Phi(x)$ and $\Pi(x)$ can be considered themselves as a family of functionals parametrized by $x \in \mathbb{R}^d$ ($ev_x^a[\Psi]$), with PB

$$\begin{aligned} \{\Phi(x), \Phi(y)\} &= 0 \ , \\ \{\Pi(x), \Pi(y)\} &= 0 \ , \\ \{\Phi(x), \Pi(y)\} &= \delta(x-y) \ . \end{aligned}$$

On noncommutative Moyal space the algebra of functions on $\mathbb{R}^d$ becomes noncommutative with noncommutativity given by the twist $\mathcal{F}$

The twist lifts to the algebra A of functionals so that this becomes noncommutative.

This is achieved by lifting to A the action of infinitesimal translations

$$\partial_i^* G := -\int \mathsf{d}^d x \; \partial_i \Phi(x) \frac{\delta G}{\delta \Phi(x)} + \partial_i \Pi(x) \frac{\delta G}{\delta \Pi(x)}$$

Therefore on functionals the twist is represented as

$$\mathcal{F} = \mathsf{e}^{-\frac{\mathsf{i}}{2}\theta^{ij} \int \mathsf{d}^d x \left(\partial_i \Phi \frac{\delta}{\delta \Phi(x)} + \partial_i \Pi \frac{\delta}{\delta \Pi(x)}\right) \otimes \int \mathsf{d}^d y \left(\partial_j \Phi \frac{\delta}{\delta \Phi(y)} + \partial_j \Pi \frac{\delta}{\delta \Pi(y)}\right)}$$

The associated $\star$-product is

$$F \star G[\Psi] = \bar{\mathsf{f}}^{\alpha}(F)[\Psi]\ \bar{\mathsf{f}}_{\alpha}(G)[\Psi]$$

If we regard $\Phi(x)$ and $\Pi(x)$ as coordinate functionals, $ev_x^a[\Psi]$, we recover the usual Moyal product

$$ev_x^a \star ev_x^b\ [\Psi] = \Psi^a \star \Psi^b\ (x) \equiv \bar{\mathsf{f}}^{\alpha}(\Psi^a)\bar{\mathsf{f}}_{\alpha}(\Psi^b)\ (x)$$

where $\Psi^a = (\Phi, \Pi)$

The action of infinitesimal translations on functionals is compatible with the twist because

$$\partial_i^*\Big(\int \mathsf{d}^d x\, \frac{\delta}{\delta\Psi^b(x)} \otimes \frac{\delta}{\delta\Psi^c(x)}\Big) = 0$$

Then we have a well defined notion of deformed Poisson bracket, $\{\ ,\ \}_\star\ :\ \mathsf{A}\otimes\mathsf{A}\rightarrow\mathsf{A}$,

$$\{F,G\}_\star := \{\bar{\mathsf{f}}^\alpha(F),\bar{\mathsf{f}}_\alpha(G)\} = \int \mathsf{d}^d x \left(\frac{\delta F}{\delta\Phi(x)}\star\frac{\delta G}{\delta\Pi(x)} - \frac{\delta G}{\delta\Pi(x)}\star\frac{\delta F}{\delta\Phi(x)}\right)$$

- This bracket satisfies

$$\begin{aligned}
\{F, G\}_\star &= -\{\bar{R}^\alpha(G), \bar{R}_\alpha(F)\}_\star \\
\{F, G \star H\}_\star &= \{F, G\}_\star \star H + \bar{R}^\alpha(G) \star \{\bar{R}_\alpha(F), H\}_\star \\
\{F, \{G, H\}_\star\}_\star &= \{\{F, G\}_\star, H\}_\star + \{\bar{R}^\alpha(G), \{\bar{R}_\alpha(F), H\}_\star\}_\star
\end{aligned}$$

In particular the $\star$-brackets among the fields are undeformed

$$\{\Phi(x), \Pi(y)\}_\star = \{\Phi(x), \Pi(y)\} = \delta(x-y) \ ,$$

$$\{\Phi(x), \Phi(y)\}_\star = \{\Phi(x), \Phi(y)\} = 0 \ ,$$

$$\{\Pi(x), \Pi(y)\}_\star = \{\Pi(x), \Pi(y)\} = 0 \ .$$

As for point mechanics, the $\star$-Poisson bracket just among coordinates is unchanged. But, for nontrivial functionals of the fields in general this is not the case because the Leibniz rule is deformed.

Poisson brackets for a, a^*

Expand Φ and Π in Fourier modes:

$$\Phi(x) = \int \frac{\mathrm{d}^d k}{(2\pi)^d \sqrt{2E_k}} \left(a(k)\, \mathrm{e}^{\mathrm{i}kx} + a^*(k)\, \mathrm{e}^{-\mathrm{i}kx} \right)$$

$$\Pi(x) = \int \frac{\mathrm{d}^d k}{(2\pi)^d} (-\mathrm{i}\hbar) \sqrt{\frac{E_k}{2}} \left(a(k)\, \mathrm{e}^{\mathrm{i}kx} - a^*(k)\, \mathrm{e}^{-\mathrm{i}kx} \right)$$

where $E_k = \sqrt{m^2 + \vec{p}^{\,2}} = \sqrt{m^2 + \hbar^2 \vec{k}^{\,2}}$, and $kx = \vec{k} \cdot \vec{x} = \sum_{i=1}^{d} k^i x^i$.

We use the usual Fourier decomposition because the free theory is undeformed.

We invert

$$a(k) \;=\; \int \mathrm{d}^d x \left(\sqrt{\frac{E_k}{2}}\, \Phi(x) + \frac{\mathrm{i}}{\hbar}\sqrt{\frac{1}{2E_k}}\, \Pi(x) \right) \mathrm{e}^{-\mathrm{i}kx}$$

$$a^*(k) \;=\; \int \mathrm{d}^d x \left(\sqrt{\frac{E_k}{2}}\, \Phi(x) - \frac{\mathrm{i}}{\hbar}\sqrt{\frac{1}{2E_k}}\, \Pi(x) \right) \mathrm{e}^{\mathrm{i}kx}$$

For each value of k, $a(k)$ and $a^*(k)$ are functionals of Φ and Π.

Therefore their $\star$-product may be computed

$$a(k)\star a(k') \;=\; \mathrm{e}^{-\frac{\mathrm{i}}{2}\theta^{ij} k_i k'_j} a(k)a(k') \quad , \qquad a^*(k)\star a^*(k') = \mathrm{e}^{-\frac{\mathrm{i}}{2}\theta^{ij} k_i k'_j} a^*(k)a^*(k') \quad ,$$

$$a^*(k)\star a(k') \;=\; \mathrm{e}^{\frac{\mathrm{i}}{2}\theta^{ij} k_i k'_j} a^*(k)a(k') \quad , \qquad a(k)\star a^*(k') = \mathrm{e}^{\frac{\mathrm{i}}{2}\theta^{ij} k_i k'_j} a(k)a^*(k') \quad ,$$

and the Poisson brackets

$$\{a(k), a^*(k')\}_\star = e^{\frac{\mathrm{i}}{2}\theta^{ij}k_i k'_j}\{a(k), a^*(k')\} = -\frac{\mathrm{i}}{\hbar}(2\pi)^d\delta(k-k') \ ,$$

The phase drops out because the delta contributes only for $k = k'$, in which case the antisymmetry of θ forces the exponent to be zero.

$$\{a(k), a(k')\}_\star = 0 \quad , \qquad \{a^*(k), a^*(k')\}_\star = 0 \ .$$

To see non trivial effects one can compute for example $\{a(k_1), a^*(k_2)\star a^*(k_3)\}_\star$

Canonical Field Quantization

Associated to the algebra A of functionals $G[\Phi, \Pi]$ there is the algebra $\widehat{\mathrm{A}}$ of functionals $\widehat{G}[\widehat{\Phi}, \widehat{\Pi}]$ on operator valued fields.

We lift the twist to $\widehat{\mathrm{A}}$ and then deform this algebra to $\widehat{\mathrm{A}}_\star$ by implementing once more the twist deformation principle.
In $\widehat{\mathrm{A}}_\star$ there is a natural notion of $\star$-commutator, according to the general prescription

$$[\ ,\ ]_\star = [\ ,\ ] \circ \mathcal{F}^{-1}\ .$$

that is

$$[\widehat{F}, \widehat{G}]_\star = [\bar{\mathsf{f}}^\alpha(\widehat{F}), \bar{\mathsf{f}}_\alpha(\widehat{G})] = \widehat{F} \star \widehat{G} - \bar{R}^\alpha(\widehat{G}) \star \bar{R}_\alpha(\widehat{F})$$

This $\star$-commutator is $\star$-antisymmetric, is a $\star$-derivation in $\hat{\mathsf{A}}_\star$ and satisfies the $\star$-Jacobi identity

$$\begin{aligned}
[\hat{F},\hat{G}]_\star &= -[\bar{R}^\alpha(\hat{G}),\bar{R}_\alpha(\hat{F})]_\star \\
[\hat{F},\hat{G}\star\hat{H}]_\star &= [\hat{F},\hat{G}]_\star\star\hat{H} + \bar{R}^\alpha(\hat{G})\star[\bar{R}_\alpha(\hat{F}),\hat{H}]_\star \\
[\hat{F},[\hat{G},\hat{H}]_\star]_\star &= [[\hat{F},\hat{G}]_\star,\hat{H}]_\star + [\bar{R}^\alpha(\hat{G}),[\bar{R}_\alpha(\hat{F}),\hat{H}]_\star]_\star
\end{aligned}$$

The commutator for coordinate fields can be computed

$$[\hat{\Phi}(x),\hat{\Pi}(y)]_\star = i\hbar\delta(x-y)\ .$$

which is in accordance with the $\star$-correspondence principle between $\star$-Poisson brackets and $\star$-commutators

Creation and annihilation operators

They are functionals of the operators $\widehat{\Phi}$, $\widehat{\Pi}$
their $\star$-commutator is evaluated using their functional dependence. We get

$$[\widehat{a}(k), \widehat{a}^\dagger(k')]_\star = (2\pi)^d \delta(k-k') \ .$$

In order to compare this expression with similar ones which have been found in the literature we make it explicit

$$\widehat{a}(k)\star\widehat{a}^\dagger(k') - \mathrm{e}^{-\mathrm{i}\theta^{ij}k'_i k_j}\widehat{a}^\dagger(k')\star\widehat{a}(k) = (2\pi)^d\delta(k-k')$$

that is the undeformed one

$$\widehat{a}(k)\widehat{a}^\dagger(k') - \widehat{a}^\dagger(k')\widehat{a}(k) = (2\pi)^d\delta(k-k')$$

In the noncommutative QFT context it appears already in Kulish.

Consistently with the $\star$-Poisson bracket, we have found that the $\star$-commutator of coordinate fields and of creation and annihilation operators are equal to the usual undeformed ones

this doesn't mean that the deformation is trivial because of the deformed Leibniz rule.

Analogous calculations in the literature based on a different definition of commutator yield a different result

$$a(k)a(k')^\dagger - \mathrm{e}^{-\frac{\mathrm{i}}{2}\theta^{ij}k_i k'_j} a(k)^\dagger a(k') = (2\pi)^d \delta(k-k')$$

Conclusions and perspectives

We have realised NC spacetime and phasespace symmetries as a NC Hopf algebra $U\Xi_\star$

by requiring compatibility with such structure we have deduced Poisson brackets and commutator relations

The canonical quantization rules are different from others in the literature which are based essentially on $U\Xi^{\mathcal{F}}$

Physical predictions should be independent on the mathematical setting ($U\Xi_\star$ and $U\Xi^{\mathcal{F}}$ are isomorphic as Hopf algebras)

We are presently applying this procedure to field theories s.t. scalar CFT and $\lambda\phi^{\star 4}$ and look into observables (stress-energy tensor, S-matrix)